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## Analysis, Control and Inverse Theory of Fluids, Waves, Materials Structures

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**07/06/2015**  
**Final Report**

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<b>12. DISTRIBUTION/AVAILABILITY STATEMENT</b> DISTRIBUTION A						
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<b>14. ABSTRACT</b> <p>The focus of this research is multi-fold: Fluids, waves, structures and their interrelationships. This research made advances on all fronts. Regarding fluid, it solved the problem of feedback stabilization of the Navier-Stokes equations by tangential controls with arbitrarily small support. Moreover, it study fluid-structure interaction models when either the structures moves within the immersed fluid; or else is static but has small and rapid oscillations. In addition, this research produced a theoretical solution to the problem of flutter suppression of a flow of gas over a panel, say an aircraft wing, at the subsonic regime, by means of inserting suitable friction on the structure only. In the supersonic regime, the problem of controlling flutter can be reduced to a finite dimensional problem after a long time. Finally, this research solved the inverse problem of recovering the electric coefficient of a Schrodinger equation defined on a Riemannian manifold (so that also the variable coefficient case is included) by means of just one boundary observation/measurement acting on a suitable part of the boundary and for an arbitrarily short time interval. here recovery means: uniqueness of the recovery and stability (wekll-posednes) of the recovery by using a boundary trace of the solution.</p>						
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During the last year of the project, July 2014- June 2015, some notable accomplishments were obtained to crown the findings of the preceding two years of the project. They include, in particular:

**A) Area concerning fluid per se, or else fluid-structure interaction**

- (i) Feedback local stabilization of the full Navier-Stokes equations in the physical dimensions 2 and 3, in a neighborhood of an equilibrium solution, by means of a pair of feedback controls  $\{u,v\}$ , acting tangentially in the way made precise below, and with arbitrarily small support. More precisely, the control  $u$  is a boundary control acting tangentially on an arbitrarily small portion of the boundary of the spatial domain. Instead, the control  $v$  is an interior control, acting likewise tangentially, that is, parallel to the boundary, and supported on a small interior domain corresponding to the boundary support of  $u$ . In conclusion, a purely tangential action is in place. The results depend drastically on the dimension. For dimension 2, the feedback controllers can be constructed explicitly as being finite dimensional. It may be either Riccati-based or else spectral based. This is not possible in the physical dimension 3. Here a Riccati-type feedback is constructed. The topological level at which decay is achieved also depends on the dimension. It is more challenging for dimension 3. The result goes through the preliminary step of establishing uniform global exponential decay for the corresponding linearized system (Oseen system). No artificial assumptions are used
- (ii) Well-posedness of the fluid-structure interaction comprised of a structure immersed in a fluid domain, and moving within the fluid domain, with coupling being active at the interface between the structure domain and the fluid domain. This implies that the overall system is quasi-linear. The results obtained about well-posedness drastically improve on the regularity obtained about ten years ago. For regular solutions, the required regularity obtained in the present AFOSR-supported research is 2 units less than in the past literature.
- (iii) Well-posedness as well as stability properties of another model of fluid-structure interaction: here the structure is instead static (save for small rapid oscillations) and is immersed in the fluid, the novelty over past literature being that the structure has an additional strong damping term, of the same order of strength as that of the principal part operator. Results include: (a) that the overall fluid-structure interaction model generates an analytic semi-group which moreover (b) is exponentially stable with a precise decay-rate. Furthermore, the resolvent set of the analytic semigroup generator contains the entire complex plane outside the negative real axis.
- (iv) Additional results to the model in part (iii) refer to the case where such fluid-structure interaction model is subject to a control acting on the interface between the two media: either in the Neumann boundary conditions of the interface conditions (the more challenging case of the analysis) or else in the Dirichlet boundary conditions of the interface conditions. The first result requires finding the domain of the square root of the matrix-valued generator, with coupled boundary conditions, a non-trivial task. As a result, this fluid-structure interaction

model is covered as a specialization of the known optimal control or min-max game theory of parabolic problems of the literature

**B) Area concerning flow-structure interaction, such as it occurs over the wing of an airplane.**

One of the fundamental problems in aero-elasticity is the control of flutter, which is induced by the oscillations of a panel immersed in the flow of gas. This could be wing of airplane traveling at a subsonic or supersonic speeds, or a fixed panel (such as a bridge), affected by the movement of the air. The flutter problem is an endemic phenomenon in technological applications. Thus, controlling flutter or altogether eliminating flutter, is one of the key goals in aero-elasticity.

The present project in reference above provided an analysis of the qualitative behavior of oscillations of a panel immersed in either a subsonic or a supersonic flow of gas. The mathematical model consists of a perturbed wave equation coupled at the interface with a nonlinear plate. The main findings are as follows:

1. In the subsonic regime:

It is proved that an application of frictional damping on the structure only (no dissipation assumed on the flow) can stabilize to equilibria turbulent flows. This is proved by asserting strong asymptotic stability. The main mathematical challenge of the problem is how to obtain the requisite estimates without accounting for any dissipative effects of the flow. Here, new techniques have been developed which allow to control the rate of stabilization to equilibria of a part of the flow. Thus the final conclusion is that the flutter can be eliminated by applying a suitable feedback control in the form of frictional damping applied on the plate.

2. In the supersonic regime:

It is proved that the flow itself has some stabilizing effect on the panel (this has also been observed experimentally). In mathematical terms, it is shown that the long time dynamics can be reduced to a finite dimensional attractor. Thus, the problem of controlling the flutter becomes finite dimensional. Since the attractor can be chaotic (it may contain limit cycles, periodic orbits, etc), controlling the ultimate behavior requires tools in finite dimensional control theory. It has been also shown that the rate of convergence to the attracting set is uniform. This result has been obtained by using theory of quasi-stability applied to delayed system. Indeed, decomposition methods allow to single out the behavior of the plate (subjected to the impact of the flow) as a nonlinear delay system. Thus, the final conclusion is that in the supersonic case the flutter problem can be reduced to a finite dimensional control problem. This is then reduced to solving an Order Reduction Problem arising in a flow-gas interaction.

3. The ultimate goal is to produce a finite dimensional control algorithm which eliminates flutter in the supersonic case. The calculations will be based on a numerical approximations of frame functions determined from the Order Reduction Algorithm.

**C) Area concerning Inverse Theory of Partial Differential Equations; more precisely, coefficient recovery**

(i) Here one start with a general Schrodinger equation, which includes electric and magnetic potential coefficients, defined on a Riemannian manifold (this case in particular includes a Schrodinger equation with space-dependent coefficients of the principal part operator). The problem is endowed with a non-homogeneous Dirichlet boundary term and an initial condition. The goal is to recover the unknown electric potential coefficient by means of just one boundary observation (measurement) to be performed on a suitable explicit support of the boundary as well as over an arbitrarily short interval. Such boundary observation will involve the Neumann-trace of the solution in the present non-homogeneous Dirichlet case. Recovery means the two canonical items of an inverse problem: uniqueness of the recovery and stability (also referred to as continuous dependence) of the recovery. The results obtained are expressed explicitly in terms of assumptions on the data (not on assumptions of the solutions as in much of the literature on inverse problems)

(ii) Another Inverse problem resolved refers to a third order (in time) PDE arising in High Intensity Ultrasound. Here again the goal is to recover (uniqueness and stability) a suitable natural coefficient by means of a boundary measurement/observation.

1.

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**Program Manager**

**The AFOSR Program Manager currently assigned to the award**

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**Abstract**

During the last year of the project, July 2014- June 2015, some notable accomplishments were obtained to crown the findings of the preceding two years of the project. They include, in particular:

A) Area concerning fluid per se, or else fluid-structure interaction

(i) Feedback stabilization of the full Navier-Stokes equations in the physical dimensions 2 and 3, in a neighborhood of an equilibrium solution, by means of a pair of feedback controls  $\{u,v\}$ , acting tangentially, and with arbitrarily small support.

(ii) Well-posedness of the fluid-structure interaction comprised of a structure immersed in a fluid domain, and moving within the fluid domain, with coupling being active at the interface between the structure domain and the fluid domain. This implies that the overall system is quasi-linear.

(iii) Well-posedness as well as stability properties of another model of fluid-structure interaction: here the structure is instead static (save for small rapid oscillations) and is immersed in the fluid, the novelty over past literature being that the structure has an additional strong damping term, of the same order of strength as that of the principal part operator.

(iv) Additional results to the model in part (iii) refer to the case where such fluid-structure interaction model is subject to a control acting on the interface between the two media: either in the Neumann boundary conditions of the interface conditions (the more challenging case of the analysis) or else in the Dirichlet

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boundary conditions of the interface conditions.

B) Area concerning flow-structure interaction, such as it occurs over the wing of an airplane.

One of the fundamental problems in aero-elasticity is the control of flutter, which is induced by the oscillations of a panel immersed in the flow of gas. One case is the wing of airplane traveling at subsonic or supersonic speeds; another case could be a fixed structure (a bridge) affected by the movement of the air. Flutter problem is an endemic phenomenon in technological applications. Thus controlling it, or altogether eliminating it, is one of the key goals. This research shows a solution to the flutter problem under either subsonic regime (by means of dissipation on the structure) or supersonic regime (where it is reduced to a finite dimensional problem)

C) Area concerning Inverse Theory of Partial Differential Equations; more precisely, coefficient recovery via just one boundary observation/measurement

We consider a general Schrodinger equation which includes electric and magnetic potential defined on a Riemannian manifold (this case in particular includes a Schrodinger equation with space-dependent coefficients of the principal part operator). The problem has a non-homogeneous Dirichlet boundary term and an initial condition. The goal is to recover the unknown electric potential coefficient by means of just one boundary observation (measurement) to be performed on an arbitrarily explicit support of the boundary as well as over an arbitrarily short interval. Such boundary observation will involve the Neumann-trace of the solution. Recovery means the two canonical items of an inverse problem: uniqueness of the recovery and stability (also referred to as continuous dependence) of the recovery. The results obtained are expressed explicitly in terms of assumptions on the data (not on assumptions of the solutions as in much of the literature on inverse problems)

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#### **Archival Publications (published) during reporting period:**

We report here only the publications during the period summer 2014- summer 2015 since the publications in the proceeding period already included in the annual report.

- S. Liu and R. Triggiani, An inverse problem for a linearized Jordan/Moore/Gibson/Thompson equation, New Prospects in Direct, Inverse, and Control Problems for Evolution Equations (Favini, et al., Eds.), Chapter 15 (2014), 305-351, Springer-INDAM Series.
- R. Triggiani and Z. Zhang , Global uniqueness and stability in determining the electric potential coefficient of an inverse problem for Schrodinger equations in Riemannian manifold, 33 pages, J. Inverse and Ill-Posed Problems, to appear.
- I. Lasiecka and R. Triggiani, Domains of fractional powers of matrix-valued operators: A general approach, invited paper, to appear in Operator Semigroups meet Complex Analysis, Harmonic Analysis and Mathematical Physics, Birkhäuser, Volume dedicated to C. Batty.

- I. Lasiecka and R. Triggiani, Uniform stabilization with arbitrary decay rates of the Oseen equation by finite-dimensional tangential localized interior and boundary controls, invited paper, Springer Proceedings in Mathematics series, Semigroup of Operators and Applications, J. Bonasiak, A. Bobrowski, M. Lachowicz, Editors, 2015, 125-154.17
- I. Lasiecka and R. Triggiani, Stabilization to an equilibrium of the Navier-Stokes equations with tangential action of feedback controllers Nonlinear Analysis, in press.
- I. Lasiecka and R. Triggiani, Heat{Structure Interaction with Viscoelastic Damping: Analyticity with Maximal Analytic Sector, Exponential Decay, 26 pp, 2015
- I. Lasiecka and R. Triggiani, A Heat{Viscoelastic Structure Interaction Model with Neumann and Dirichlet Boundary Control at the Interface: Regularity, Optimal Control, and Min-Max Game Theory, 22 pp, 2015
- I. Chueshov, I. Lasiecka and J. Webster, Flow-Plate Interactions: Well-posedness and Long-Time Behavior - accepted for Discrete and Continuous Dynamical Systems , Vol 7, Nr 5, (2014), pp. 925-965
- M. Cavalcanti, Valeria Domingos-Cavalcanti, I. Lasiecka and F. Nascimento, Intrinsic decay rate estimates for the wave equation with competing viscoelastic and frictional damping that is localized, , Discrete and Continuous Dynamical Systems-B , vol 19, Nr 7, pp 1987-2012, 2014
- I. Lasiecka and J. P. Graber Analyticity and Gevrey class regularity for a strongly damped wave equation with hyperbolic boundary conditions, Semigroup Forum , vol 88, pp 333-365, 2014
- I. Lasiecka and N. Fourrier, Regularity and stability of a wave equation with strong damping and dynamic boundary conditions, Evolution Equations and Control Theory , Vol 2, Nr 4, pp 631-667, 2013.
- M. Cavalcanti, V. Cavalcanti, I. Lasiecka, F. Nascimento and J. Rodrigues, Uniform decay rates for the energy of Timoshenko system with arbitrary speeds of propagation and localized nonlinear damping., to appear ZAMP
- M. Ignatova, I. Kukavica, I. Lasiecka and A. Tuffaha, On wellposedness and small data global existence for an interface damped free boundary fluid-structure interaction model - Nonlinearity , vol 27, issue 3, pp 467-499, 2014
- I. Lasiecka and J. Webster Nonlinear Plates Interacting with A Subsonic, Inviscid Flow via Kutta-Joukowski Interface Conditions-, Nonlinear Analysis: Real World Applications. vol 17, pp 171-191, 2014,
- I. Chueshov, I. Lasiecka and J. Webster, Attractors for Delayed, Non-Rotational von Karman Plates with Applications to Flow-Structure Interactions Without any Damping to appear Communications on Partial Differential Equations
- I. Lasiecka and J. Webster, Eliminating flutter for clamped von Karman plates immersed in subsonic flows, Communications on Pure and Applied Analysis} , special volume dedicated to M. Vishik. Vol 13 (5), pp 1935-1969, 2014
- M. Cavalcanti, A. Delano, I. Lasiecka and Xiaojun Wang, Existence and Sharp Decay Rate Estimates for a von Karman System with Long Memory -to appear Nonlinear Analysis Series B: Real World Applications

**Changes in research objectives (if any):**

not applicable

**Change in AFOSR Program Manager, if any:**

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**Extensions granted or milestones slipped, if any:**

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